USN | | | | | | | | | | |

06CS34

Third Semester B.E. Degree Examination, December 2010
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Determine the sets A and B, given that $A - B = \{1, 3, 7, 11\}$, $B - A = \{2, 6, 8\}$ and $A \cap B = \{4, 9\}$. **(04 Marks)**

b. Using the Venn diagram, prove that $A \Delta (B \Delta C) = (A \Delta B) \Delta C$. **(05 Marks)**

c. A professor has two dozen introductory text books on computer science and is concerned about their coverage of the topics (A) Compilers, (B) Data structures and (C) Operating systems. The following data are the number of books that contain material on these topics: $|A| = 8$, $|B| = 13$, $|C| = 13$, $|A \cap B| = 5$, $|A \cap C| = 3$, $|B \cap C| = 6$, $|A \cap B \cap C| = 2$
i) How many of the text books include material on exactly one of these topics?
ii) How many do not deal with any of the topics?
iii) How many have no materials on compilers? **(06 Marks)**

d. Show that (0, 1) is an uncountable set. **(05 Marks)**

2 a. By constructing truth tables, show that $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$. **(05 Marks)**

b. Define tautology. Prove that $[(P \vee Q) \wedge \neg\{\neg P \wedge (\neg Q \vee \neg R)\}] \vee (\neg P \wedge \neg Q) \vee ((\neg P \wedge \neg R)$ is a tautology without using truth tables. **(05 Marks)**

c. Define the dual of a logical statement. Write the dual of $(P \vee T_0) \wedge (q \vee F_0) \wedge (r \wedge s \wedge T_0)$. **(04 Marks)**

d. Test the validity of the following argument:
i) If I study, I will not fail in the examination.
ii) If I do not watch TV in the evenings, I will study.
iii) I failed in the examination.
iv) Therefore, I must have watched TV in the evenings. **(06 Marks)**

3 a. Define: i) Open sentence ii) Quantifiers.
Write the following propositions in symbolic form and find its negation:
"All integers are rational numbers and some rational numbers are not integers". **(07 Marks)**

b. Let $p(x) : x \geq 0$, $q(x) : x^2 \geq 0$ and $r(x) : x^2 - 3x - 4 = 0$. Then for the universe comprising of all the real numbers, find the truth values of
i) $\exists x, [p(x) \wedge q(x)]$,
ii) $\forall x, [p(x) \rightarrow q(x)]$ and
iii) $\exists x, [p(x) \wedge r(x)]$. **(06 Marks)**

c. Give i) a direct proof, ii) an indirect proof for the following statement:
"If n is an odd integer, then (n + 9) is an even integer". **(07 Marks)**

4 a. For all the positive integers n , prove that if $n \geq 24$, then n can be written as a sum of 5's and / or 7's. **(07 Marks)**

b. If $F_0, F_1, F_2, \ldots\ldots\ldots\ldots$ are Fibonacci numbers, prove that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$. **(07 Marks)**

c. A sequence $\{ a_n \}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$, for $n \geq 2$. Find a_n in explicit form. **(06 Marks)**

PART – B

5 a. Define the Cartesian product of two sets. For any three non-empty sets A, B and C, prove that $A \times (B - C) = (A \times B) - (A \times C)$ **(05 Marks)**

b. Shirts numbered consecutively from 1 to 20 are worn by 20 students of a class. When any 3 of these students are chosen to be a debating team from the class, the sum of their shirt numbers is used as the code number of the team. Show that if any 8 of the 20 are selected, then from these 8 we may form at least two different teams having the same code number. **(05 Marks)**

c. i) Let A and B be finite sets with $| A | = m$ and $| B | = n$. Find how many functions are possible from A and B?

ii) If there are 2187 functions from A to B and $| B | = 3$, what is $| A |$? **(05 Marks)**

d. Let $A = B = C = R$ and $f : A \rightarrow B$ and $g : B \rightarrow C$ be defined by $f(a) = 2a + 1$, $g(b) = ⅓\, b$, $\forall a \in A$, $\forall b \in B$. Compute gof and show that gof is invertible. What is $(gof)^{-1}$? **(05 Marks)**

6 a. Let A = { 1, 2, 3, 4, 6 } and R be a relation on A defined by aRb if and only if "a is a multiple of b". Write down the relation matrix M(R) and draw its digraph. **(06 Marks)**

b. In the following problems, consider the partial order of divisibility on the set A. Draw the Hasse diagram of the poset and determine whether the poset is linearly ordered (totally ordered) or not.

i) A = { 1, 2, 3, 5, 6, 10, 15, 30 } ii) A = { 2, 4, 8, 16, 32 } **(07 Marks)**

c. Let A = { 1, 2, 3, 4, 5 } × { 1, 2, 3, 4, 5 }. Define a relation R on A by $(x_1, y_1)\ R\ (x_2, y_2)$ iff $x_1 + y_1 = y_2 + x_2$.

i) Verify that R is an equivalence relation on R.

ii) Determine the partition of A induced by R. **(07 Marks)**

7 a. Define a group, with an example. **(06 Marks)**

b. State and prove the Lagrange's theorem. **(06 Marks)**

c. Define the homomorphism and isomorphism in a group. Let f be homomorphism from a group G_1 to a group G_2. Prove that

i) If e_1 is the identity in G_1 and e_2 is the identity in G_2 , then $f(e_1) = e_2$.

ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$. **(08 Marks)**

8 a. For all $x, y \in Z_2^n$, prove that $wt(x + y) \leq wt(x) + wt(y)$. **(04 Marks)**

b. Let $E : Z_2^m \rightarrow Z_2^n$, $m < n$ be an encoding function given by a generator matrix G or the associated parity check matrix H. Prove that $C = E(Z_2^m)$ is a group code. **(08 Marks)**

c. Let u = { 1, 2 } and R = P(u). Define '+' '.' on the elements of R by $A + B = A \Delta B$, $A \cdot B = A \cap B$. Prove that R is a commutative ring with unity but not an integral domain. **(08 Marks)**

* * * * *